Quiz 3 on Fri Nov 17:

Oct 27 – Nov 10

- Exponential functions $\leftrightarrows$ their derivatives
- Inverse functions, logarithms
- Differential Equation - $y' = ky$, solve w/ init. condition

Be able to
- Check solutions $y(t)$.

- Autonomous DE's: qualitative analysis

Draw/interpret phase line
slope field
steady states
stability & long term behavior of trajectories

- Linear DE's (today)

$$y' = a - by \quad a, b \text{ constants}$$

Object in environment (Clicker Q1)

- $\underbrace{T(t) = E}$; and when $T(t) = E$, $T'(t) = 0$.

stable steady state.

$k(E - T(t))$ & $\underline{k > 0}$ b/c $T(t)$ moves towards E.

constant diff. b/w temperatures

$$T' = k(E - T)$$

$$T' = \underbrace{kE}_{a} - \underbrace{k}_{b}T$$

Solving $y' = a - by$

$$= -b\left(y - \frac{a}{b}\right)$$

$y = \frac{a}{b}$ is a steady state.

Linear sub:

plug in

Let $\begin{cases} z(t) = y(t) - \frac{a}{b} \\ z'(t) = y'(t) \end{cases}$

$$z' = -b \cdot z$$

(Ex. y = temp in °K
z = temp in °C)

Solution: $z(t) = Ce^{-bt}$

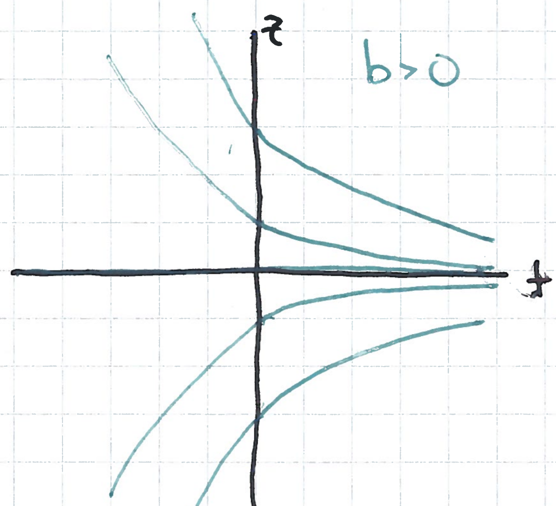
(C any constant)

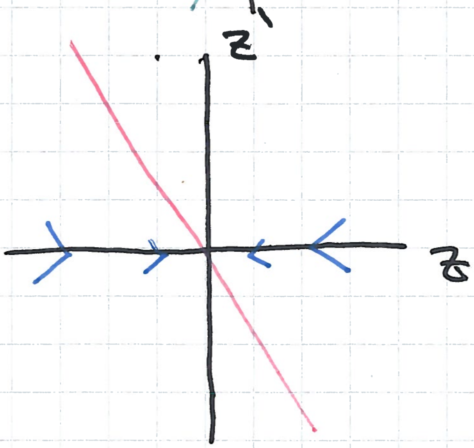
Substitute back

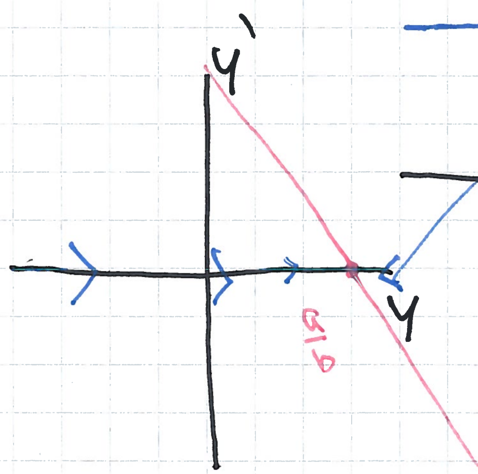
$$z(t) = y(t) - \frac{a}{b}$$

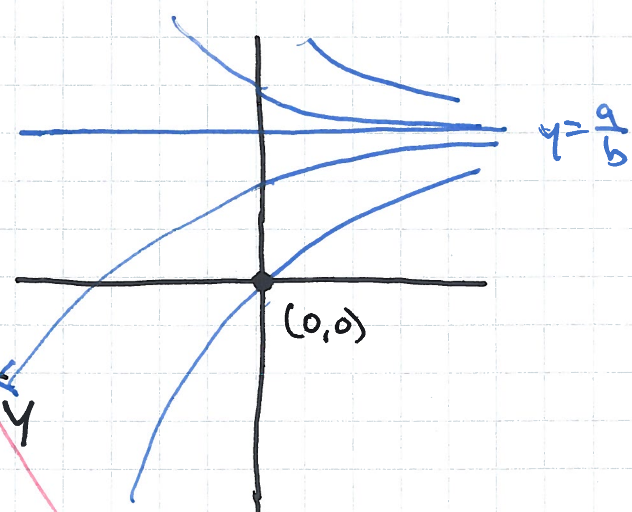
$$Ce^{-bt} = y(t) - \frac{a}{b}$$

$$\boxed{y(t) = \frac{a}{b} + Ce^{-bt}}$$









Ex: Initial condition $y(0)=0$

$$0 = y(0) = \frac{a}{b} + C\cdot e^{-b\cdot 0}$$

$$= \frac{a}{b} + C \quad \rightsquigarrow \quad C = -\frac{a}{b}.$$

Thus, $y(t) = \frac{a}{b} - \underbrace{\frac{a}{b}e^{-bt}}_{\text{as } t\to\infty \text{, this term shrinks to } 0} = \frac{a}{b}(1-e^{-bt})$

In general, substitution is a useful technique.

Ex: $z(t) = e^{y(t)}$ ends up being useful.

Drug Delivery (Clicker Q2)

$$d'(t) = \text{Drug in} - \text{Drug out}$$

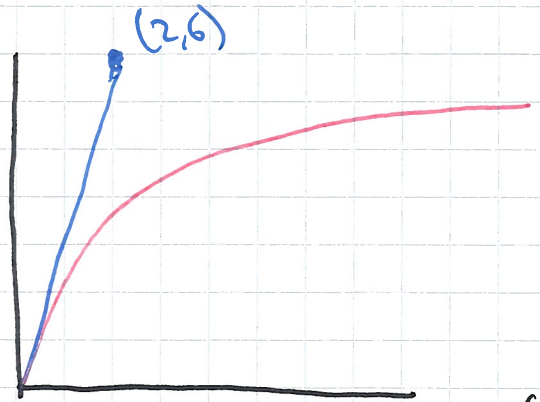
$$\overset{\text{Drug in}}{=} k_{IV} - \overset{\text{Drug out}}{=} k_m d$$

"target dose"

$$d' = \underbrace{k_{IV}}_{a} - \underbrace{k_m}_{b} d \rightsquigarrow d(t) = \boxed{\frac{k_{IV}}{k_m}} + \underbrace{Ce^{-k_m t}}_{\text{as } t\to\infty\text{, this} \to 0}$$

$$= -k_m\left(d - \frac{k_{IV}}{k_m}\right)$$

If $d(0)=0$, $d(t) = \frac{k_{IV}}{k_m}\left(1-e^{-k_m t}\right)$

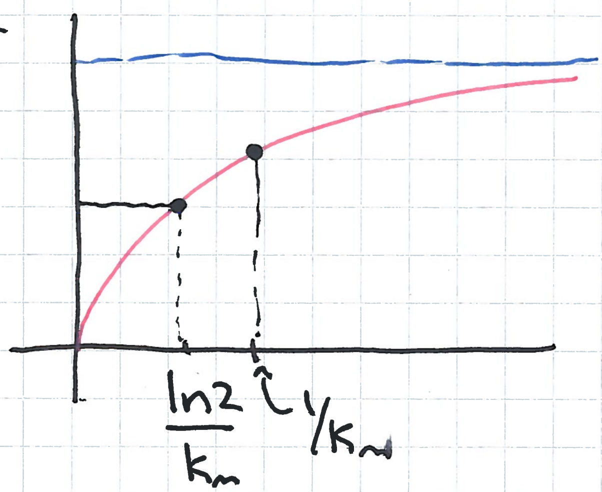


k_{IV} is the slope at $t=0$.

$d'(0) = 0$

$\left(\text{given } d(t) = \frac{k_{IV}}{k_m}\left(1-e^{-k_m t}\right)\right)$

Q4



$d(t) = \frac{1}{2}\,\frac{k_{IV}}{k_m}$

$\frac{k_{IV}}{\cancel{k_m}}\left(1-e^{-k_m t}\right) = \frac{1}{2}\,\frac{k_{IV}}{\cancel{k_m}}$

$1 - e^{-k_m t} = \frac{1}{2}$

$\frac{1}{2} = e^{-k_m t}$

$\frac{1}{k_m}$ = time req to reach $1-\frac{1}{e}$ of dose

$-\ln 2 = \ln\frac{1}{2} = -k_m t$

= "characteristic time"

$$\boxed{\frac{\ln 2}{k_m} = t}$$

As k_m increases, the time req to reach $\frac{1}{2}$ decreases.